

NAME: _____
(Print) Last Name, First Name

STUDENT NUMBER: _____



The Irving K. Barber School of Arts and Sciences

Physics 232— Winter 2013/2014 – Term 2
FINAL EXAMINATION

Instructor: Jake Bobowski

Monday, April 28, 2014 Time: 09:00 - 12:00
Location: ART 214

This Examination was prepared by Jake Bobowski
Not including this coversheet, the exam consists of 16 numbered pages.

- Attempt any 7 of the 8 problems.
- Put a cross through one of numbers that are listed just below to indicate the problem that you do not wish to be graded

1 2 3 4 5 6 7 8

If necessary, you may use the backs of pages for calculations.

You must clearly show your work to receive full credit.

Writing down only the correct final answer will not earn full credit.

1	2	3	4	5	6	7	8	total
10	10	10	10	10	10	10	10	70

Free Response: Write out complete answers to the following questions. Show your work.

- (10^{pts}) 1. Consider the periodic function $f(x) = |\sin x|$ which, on the interval $-\pi < x < \pi$ can be expressed as:

$$f(x) = \begin{cases} -\sin x, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$$

(a) Sketch several periods of $f(x)$. Be sure to include scales for both the x - and y -axes of your plot. (**2 marks**)

(b) Find the Fourier series for this function. Simplify your answers as much as possible. Write out the first five non-zero terms of the Fourier series. (**8 marks**)

You may make use the following relations:

$$\int_{-\pi}^0 \sin x \cos nx \, dx = - \int_0^{\pi} \sin x \cos nx \, dx = \frac{\cos n\pi + 1}{n^2 - 1}$$
$$\int_{-\pi}^0 \sin x \sin nx \, dx = \int_0^{\pi} \sin x \sin nx \, dx = - \frac{\sin n\pi}{n^2 - 1}$$

which are valid for $n \neq 1$ and the following trigonometric identities:

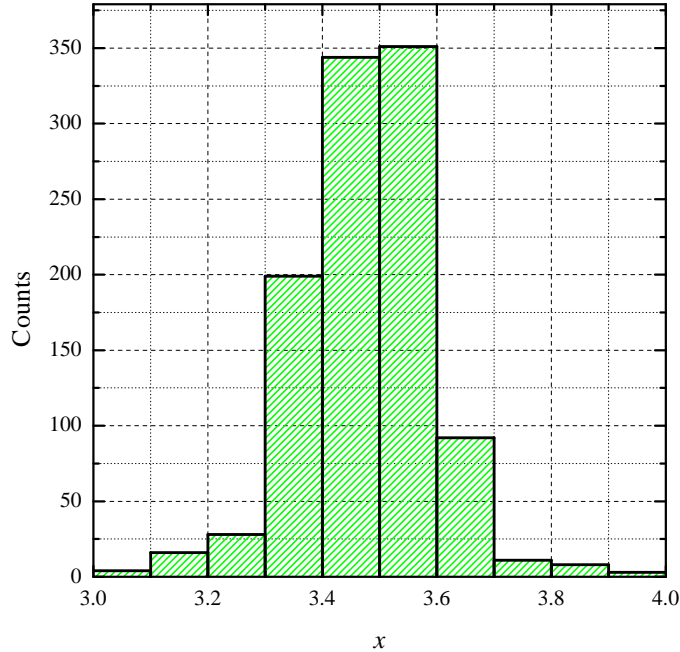
$$\sin x \cos x = \frac{1}{2} \sin 2x$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

- (10^{pts}) **2.** Because statistically there is a certain number of people that will fail to show up for flights, airlines routinely overbook their flights. Assume that there is a 3% chance that any given passenger booked on a flight will be a no-show.
- (a) For a particular flight from Vancouver to Winnipeg the plane has a 100 person capacity. Indicate which of the Gaussian, binomial, or Poisson distributions accurately describes the probability distribution of no-shows for the flight? (List all of the valid choices, there may be more than one). (**2 marks**)
- (b) If the airline sells 3 extra tickets for the flight, what is the probability that the flight will be overbooked? (**5 marks**)
- (c) If this flight is offered 10 times, what is the probability that it is overbooked at least 3 times? (**3 marks**)

(10^{pts}) **3.** The Gaussian distribution is given by:

$$P_G(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right]$$

The figure below shows some data that are approximately Gaussian distributed.



(a) What is the numerical value of $P_G(x = \mu \pm \sigma)/P_G(x = \mu)$? Show your work. **(2 marks)**

(b) Estimate the value of μ and σ of the distribution shown in the figure above. Draw the approximate shape of the Gaussian distribution that the data in the histogram follow. **(4 marks)**

(c) The distribution in the figure is made up of $N = 1061$ individual measurements. What is the approximate error in the mean value estimated from the distribution shown above? **(4 marks)**

- (10^{pts}) 4. You have three resistors with specified resistances and uncertainties: $R_1 \pm \sigma_1$, $R_2 \pm \sigma_2$, and $R_3 \pm \sigma_3$.

(a) If the three resistors are connected in series, the equivalent resistance is given by:

$$R_s = R_1 + R_2 + R_3$$

Find an expression for the uncertainty in R_s (σ_s) in terms of R_1 , R_2 , R_3 and their uncertainties. (**2 marks**)

(b) If the three resistors are connected in parallel, the equivalent resistance is given by:

$$R_p = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

Find an expression for the uncertainty in R_p (σ_p) in terms of R_1 , R_2 , R_3 and their uncertainties. (**5 marks**)

(c) Suppose you want to make a 300Ω resistor. Given the limited equipment that you have in the lab, your options are to combine three $100 \Omega \pm 5\%$ resistors in series or to combine three $900 \Omega \pm 5\%$ resistors in parallel. Compare the resulting numerical values of σ_s and σ_p . (**3 marks**)

- (10^{pts}) 5. In an experiment to measure the work function W of tungsten, one measures the electron current I as a function of the tungsten temperature T . Theoretically, these variables are expected to be related by the equation:

$$\frac{I}{A} = BT^2 \exp\left(\frac{-W}{k_B T}\right)$$

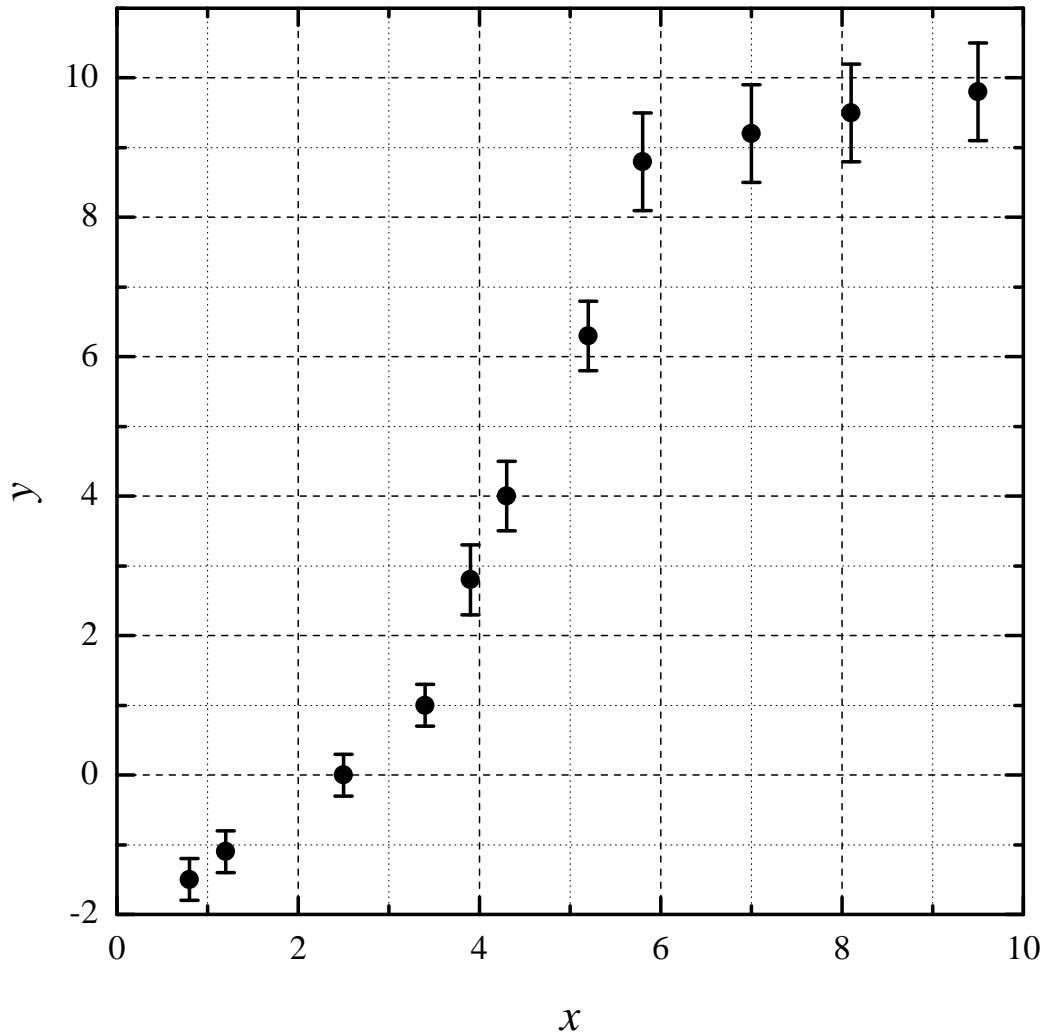
where A is the surface area of the tungsten sample, k_B is Boltzmann's constant, and B is a constant. Assume that Boltzmann's constant is known to within some uncertainty σ_{k_B} (i.e., $k_B \pm \sigma_{k_B}$ is known). However, B is an unknown constant and the tungsten sample is an odd shape so that its surface area A is also unknown.

- (a) Linearize the equation above such that the work function W can be extracted from the slope of a straight line. Give the equation of the straight line and describe the plot (y vs x) that you would generate. What does y represent and what does x represent? (**7 marks**)
- (b) If slope of your graph and its uncertainty ($m \pm \sigma_m$) are determined from a linear fit, how would you determine the uncertainty in the work function σ_W ? (**3 marks**)

0 pts

(10^{pts}) **6.** An experimental physicist has collected data $y \pm \sigma_y$ versus $x \pm \sigma_x$ as shown in the table below and as plotted in the figure below. In the plot only the y -error bars are shown.

x	σ_x	y	σ_y
0.8	0.4	-1.5	0.3
1.2	0.4	-1.1	0.3
2.5	0.4	0.0	0.3
3.4	0.4	1.0	0.3
3.9	0.4	2.8	0.5
4.3	0.4	4.0	0.5
5.2	0.4	6.3	0.5
5.8	0.4	8.8	0.7
7.0	0.4	9.2	0.7
8.1	0.4	9.5	0.7
9.5	0.4	9.8	0.7

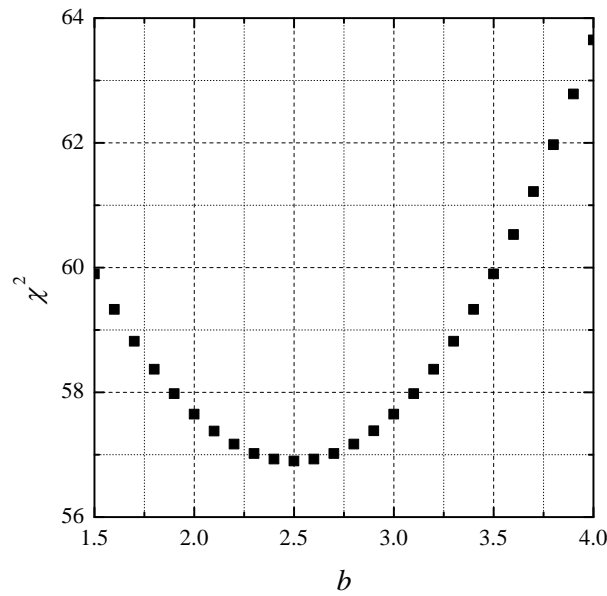


The physicist has a theoretical model that she wants to fit to the data and she wants the fit to be a *weighted* fit. She thinks that for this dataset the errors in the x measurements are not negligible and wants to include their contributions while using the standard weighted fit procedure.

(a) Describe in a few short sentences the procedure that can be used to include the contributions of the σ_x uncertainties when doing the weighted fit. (**3 marks**)

(b) When completing a weighted fit, the weighting factor used for data point $(x_i \pm \sigma_{x,i}, y_i \pm \sigma_{y,i})$ is $1/\sigma_{\text{net},i}^2$. Determine σ_{net} for the three points $(1.2 \pm 0.4, -1.1 \pm 0.3)$, $(4.3 \pm 0.4, 4.0 \pm 0.5)$, and $(8.1 \pm 0.4, 9.5 \pm 0.7)$. (**7 marks**)

- (10^{pts}) **7.** Suppose that a quantity $y(x)$ as been measured as a function of x . Theoretically, y is expected to depend on x and also on a set of parameters a , b , and c such that $y = y(x; a, b, c)$. The exact form of y does not matter for this problem, but as an example, the function could be $y = a \sin(x/b) + c$. The data are fit to the model and the best-fit parameters of a , b , and c are determined. The experimenter next tries to estimate the uncertainty in each of the parameters. In the figure below, parameters a and c are fixed at their best-fit values and the calculated χ^2 is shown as a function of b .

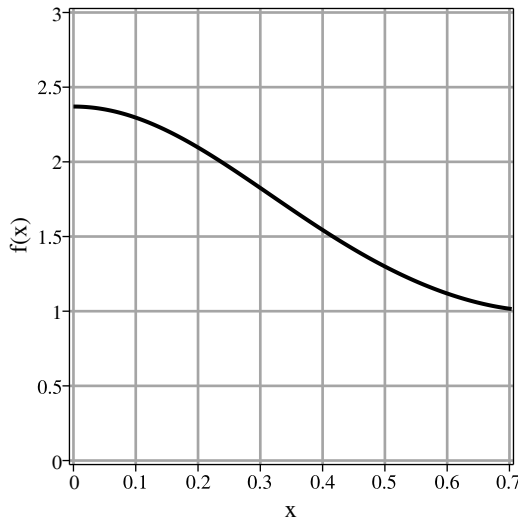


- (a) Write down the general expression for χ^2 . **(2 marks)**
- (b) When collecting the y versus x dataset N measurements were collected and reasonable estimates of σ_y were made. Based on the plot above, estimate the value of N . Explain your reasoning. **(3 marks)**
- (c) Based on the plot above, estimate the best-fit value of b and its uncertainty σ_b . Explain your reasoning. **(5 marks)**

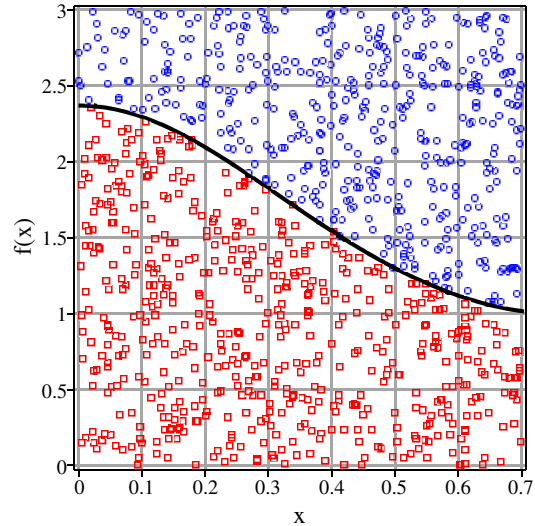
(10^{pts}) **8.** In this problem you will attempt to estimate the value of the following definite integral:

$$I = \int_0^{0.7} \frac{dx}{\sin^5(1+x^2)}.$$

This integral is not easily evaluated analytically, so the Monte Carlo “Hit & Miss” method will be used to numerically estimate I .



(i)



(ii)

(a) Briefly describe how the Monte Carlo Hit & Miss method is used to estimate the values of definite integrals. (**3 marks**)

(b) In figure (i) above the function $f(x) = \sin^{-5}(1+x^2)$ is plotted over the interval $0 \leq x \leq 0.7$. Figure (ii) shows an implementation of the hit & miss method. $N = 1000$ trials were attempted and they are all shown in the figure. The number of hits (red squares) recorded was $Z_N = 558$. What is the numerically determined value of I from this simulation? (**4 marks**)

(c) Estimate the uncertainty in the determination of I . (**3 marks**)

